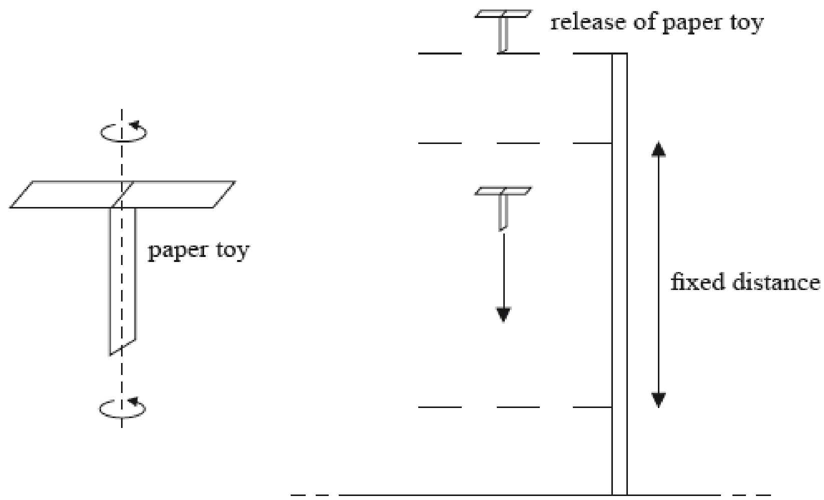
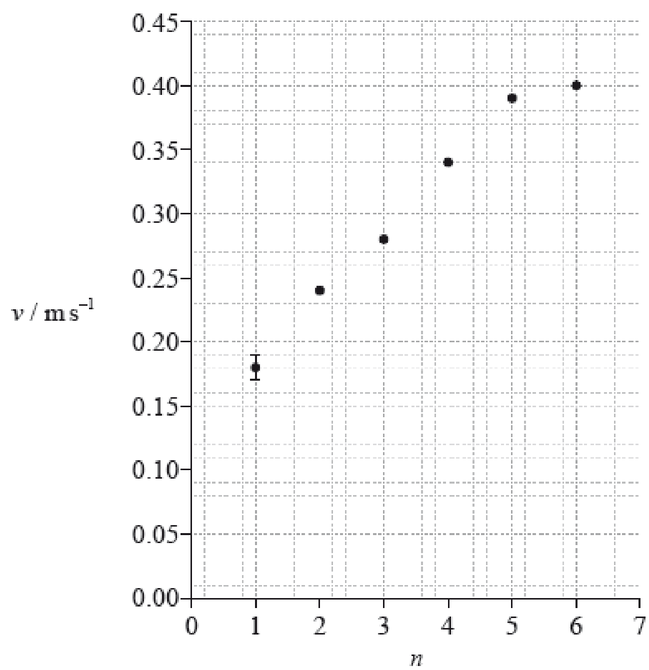
SL Paper 2

A student performs an experiment with a paper toy that rotates as it falls slowly through the air. After release, the paper toy quickly attains a constant vertical speed as measured over a fixed vertical distance.



The aim of the experiment was to find how the terminal speed of the paper toy varies with its weight. The weight of the paper toy was changed by using different numbers of paper sheets in its construction.

The graph shows a plot of the terminal speed v of the paper toy (calculated from the raw data) and the number of paper sheets n used to construct the toy. The uncertainty in v for $n = 1$ is shown by the error bar.



The fixed distance is 0.75 m and has an absolute uncertainty of 0.01 m. The percentage uncertainty in the time taken to fall through the fixed distance is 5%.

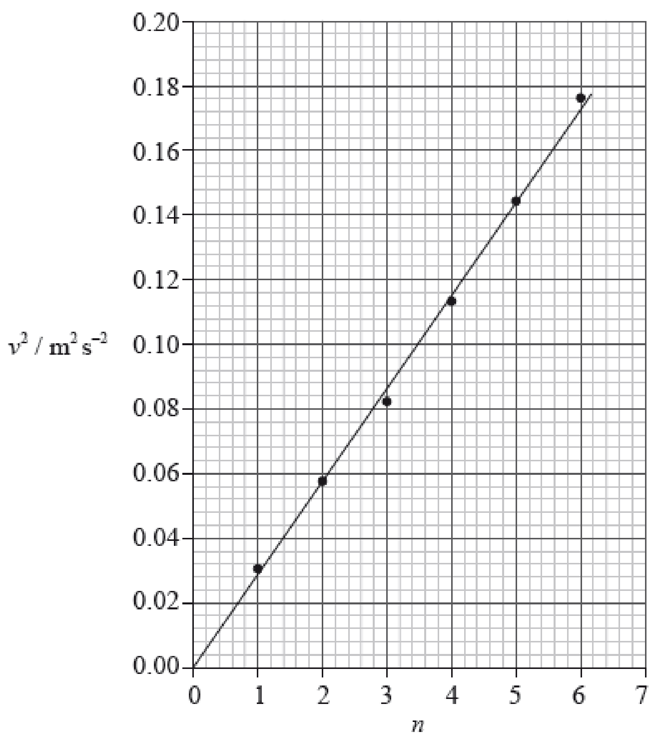
a.i. Calculate the absolute uncertainty in the terminal speed of the paper toy for $n = 6$. [3]

a.ii. On the graph, draw an error bar on the point corresponding to $n = 6$. [1]

b. On the graph, draw a line of best-fit for the data points. [1]

c. The student hypothesizes that *v* is proportional to *n*. Use the data points for $n = 2$ and $n = 4$ from the graph opposite to show that this hypothesis is incorrect. [3]

d. Another student hypothesized that v might be proportional to n. To verify this hypothesis he plotted a graph of v^2 against $\sqrt{n}$ as shown below. [3]

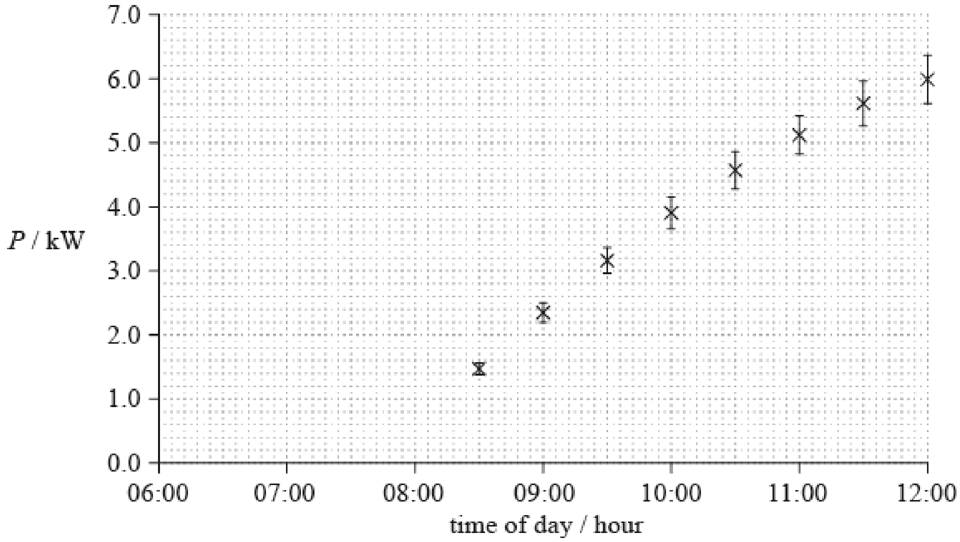


Explain how the graph verifies the hypothesis that v is proportional to $\sqrt{n}$.

Data analysis question.

An array of photovoltaic cells is used to provide electrical energy for a house. When the array produces more power than is consumed in the house, the excess power is fed back into the mains electrical supply for use by other consumers.

The graph shows how the power *P* produced by the array varies with the time of day. The error bars show the uncertainty in the power supplied. The uncertainty in the time is too small to be shown.

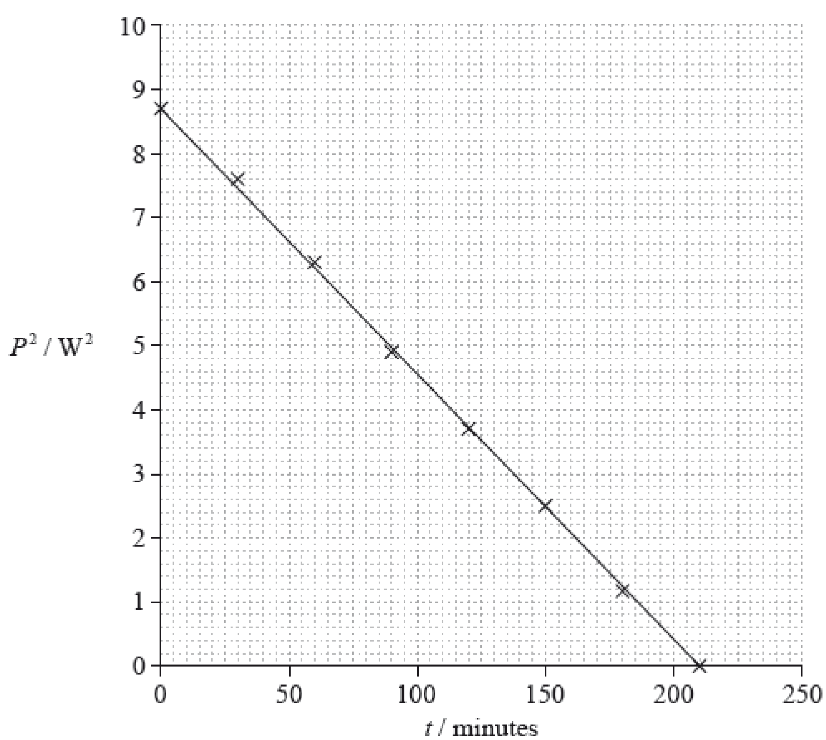


a. Using the graph, estimate the time of day at which the array begins to generate energy. [2]

b. The average power consumed in the house between 08:00 and 12:00 is 2.0 kW. Determine the energy supplied by the array to the mains electrical supply between 08:00 and 12:00. [3]

c. The power *P* produced by the array is calculated from the generated emf *V* and the fixed resistance *R* of the array using the equation $\frac{V^2}{R}$. The uncertainty in the value of *R* is 2%. Calculate the percentage uncertainty in *V* at 12:00. [3]

d. Later that day a second set of data was collected starting at $t = 0$. The variation of P^2 with time t since the start of this second data collection is shown in the graph. [3]

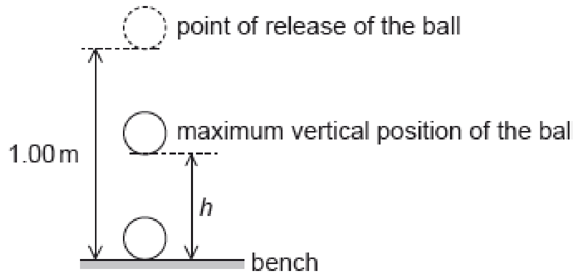


Using the graph, determine the relationship between P^2 and t.

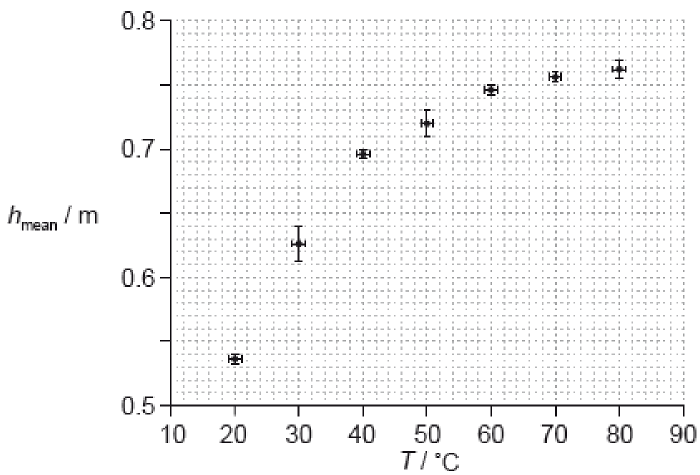
Data analysis question.

An experiment is undertaken to investigate the relationship between the temperature of a ball and the height of its first bounce.

A ball is placed in a beaker of water until the ball and the water are at the same temperature. The ball is released from a height of 1.00 m above a bench. The maximum vertical height h from the bottom of the ball above the bench is measured for the first bounce. This procedure is repeated twice and an average h_{mean} is calculated from the three measurements.



The procedure is repeated for a range of temperatures. The graph shows the variation of h_{mean} with temperature T.



a. Draw the line of best-fit for the data. [1]

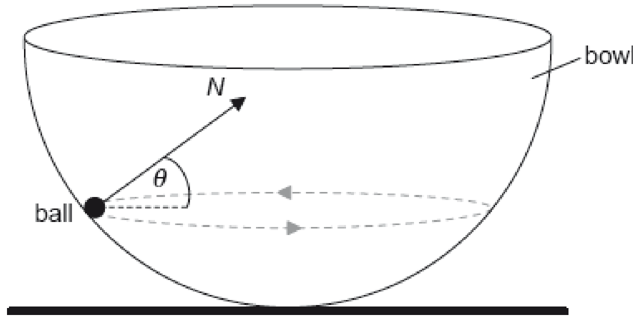
b. State why the line of best-fit suggests that h_{mean} is not proportional to T. [1]

c.i. State the uncertainty in each value of T. [1]

c.ii. The temperature is measured using a liquid in glass thermometer. State what physical characteristic of the thermometer suggests that the change in the liquid's length is proportional to the change in temperature. [1]

d. Another hypothesis is that $h_{\mathrm{mean}} = KT^3$ where K is a constant. Using the graph on page 2, calculate the absolute uncertainty in K corresponding to $T = 50°\mathrm{C}$. [4]

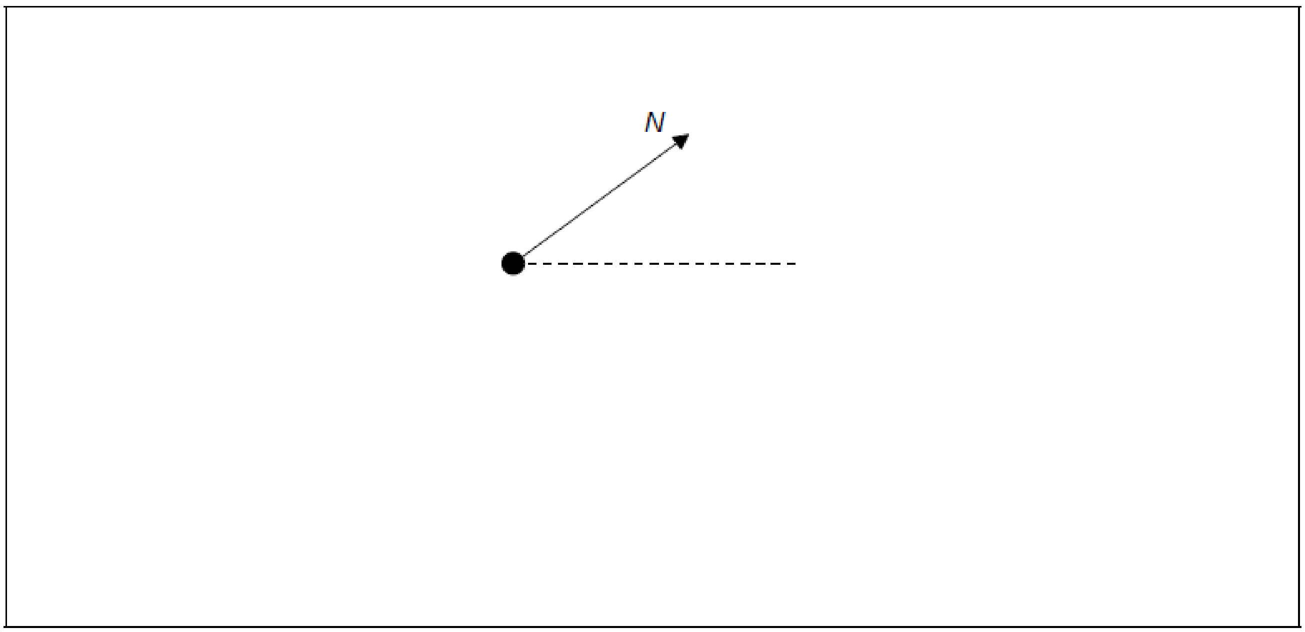
A small ball of mass *m* is moving in a horizontal circle on the inside surface of a frictionless hemispherical bowl.



The normal reaction force N makes an angle θ to the horizontal.

a.i. State the direction of the resultant force on the ball. [1]

a.ii. On the diagram, construct an arrow of the correct length to represent the weight of the ball. [2]



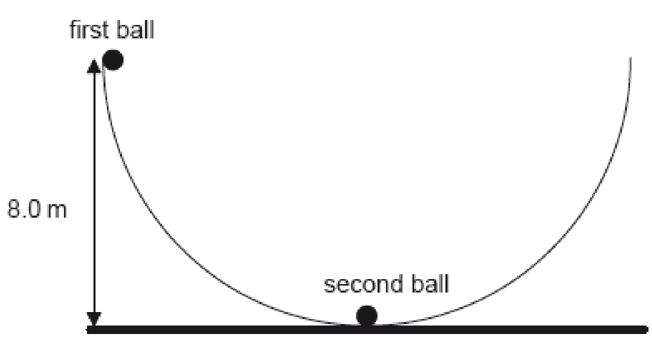
a.iii. Show that the magnitude of the net force F on the ball is given by the following equation. [3]

$$F = \frac{mg}{\tan\theta}$$

b. The radius of the bowl is 8.0 m and θ = 22°. Determine the speed of the ball. [4]

c. Outline whether this ball can move on a horizontal circular path of radius equal to the radius of the bowl. [2]

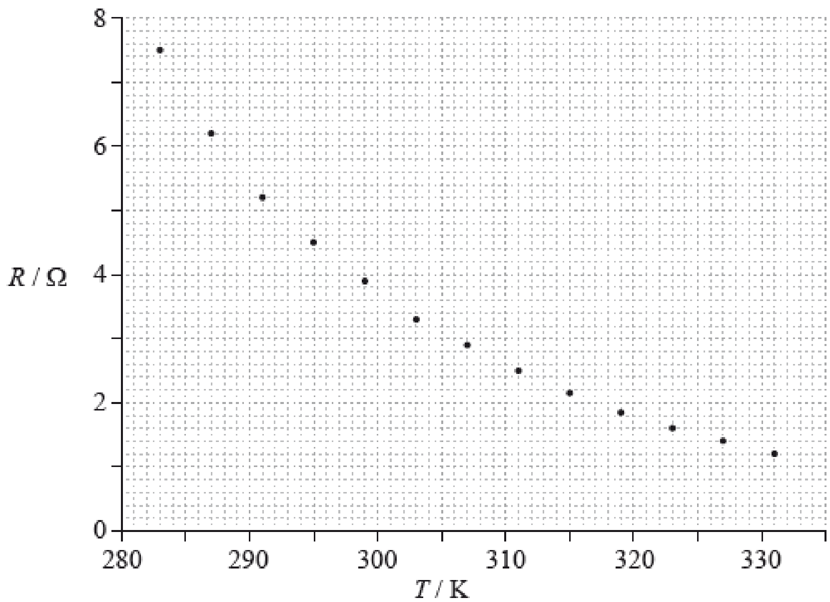
d. A second identical ball is placed at the bottom of the bowl and the first ball is displaced so that its height from the horizontal is equal to 8.0 m. [3]



The first ball is released and eventually strikes the second ball. The two balls remain in contact. Determine, in m, the maximum height reached by the two balls.

Data analysis question.

A student sets up a circuit to study the variation of resistance R of a negative temperature coefficient (NTC) thermistor with temperature T. The data are shown plotted on the graph.



The electric current through the thermistor for $T = 283\ \mathrm{K}$ is 0.78 mA. The uncertainty in the electric current is 0.01 mA.

a. Draw the best-fit line for the data points. [1]

b.i. Calculate the gradient of the graph when $T = 291\ \mathrm{K}$. [3]

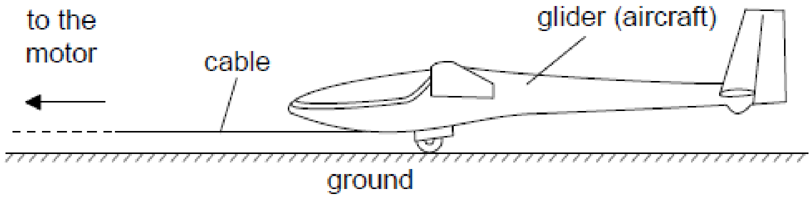
b.ii. State the unit for your answer to (b)(i). [1]

c. The uncertainty in the resistance value is 5%. The uncertainty in the temperature is negligible. On the graph, draw error bars for the data point at $T = 283\ \mathrm{K}$ and for the data point at $T = 319\ \mathrm{K}$. [2]

d.i. Calculate the power dissipated by the thermistor at $T = 283\ \mathrm{K}$. [1]

d.ii. Determine the uncertainty in the power dissipated by the thermistor at $T = 283\ \mathrm{K}$. [3]

A glider is an aircraft with no engine. To be launched, a glider is uniformly accelerated from rest by a cable pulled by a motor that exerts a horizontal force on the glider throughout the launch.



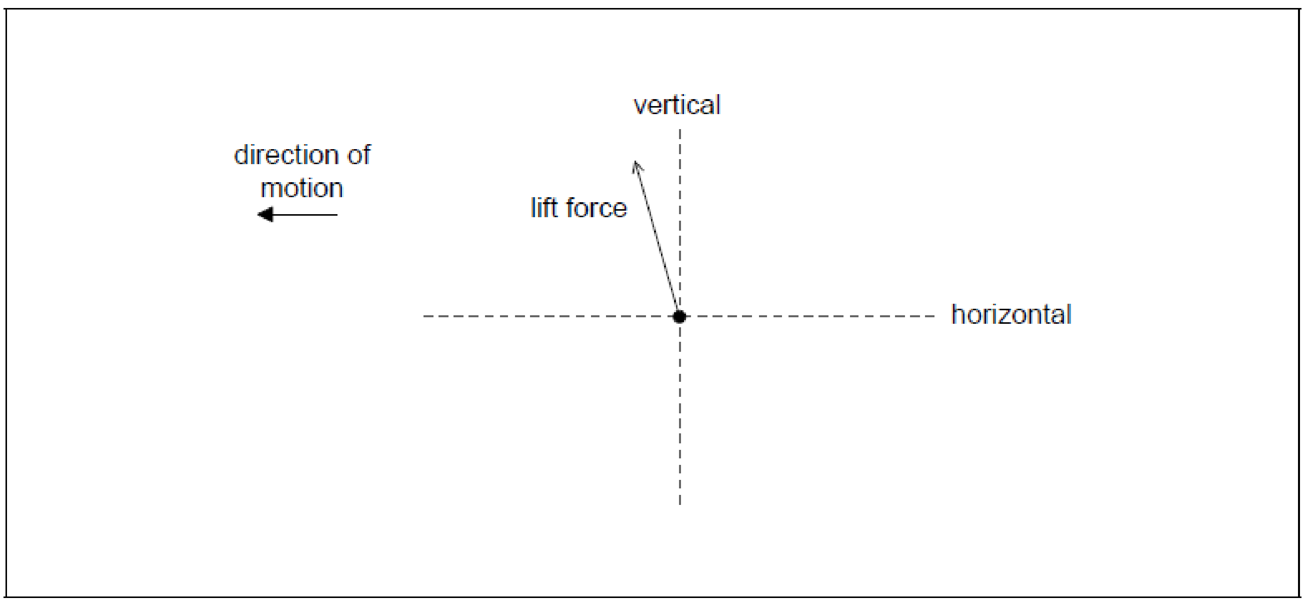
a. The glider reaches its launch speed of 27.0 m s^{-1} after accelerating for 11.0 s. Assume that the glider moves horizontally until it leaves the ground. Calculate the total distance travelled by the glider before it leaves the ground. [2]

b. The glider and pilot have a total mass of 492 kg. During the acceleration the glider is subject to an average resistive force of 160 N. Determine the average tension in the cable as the glider accelerates. [3]

c. The cable is pulled by an electric motor. The motor has an overall efficiency of 23 %. Determine the average power input to the motor. [3]

d. The cable is wound onto a cylinder of diameter 1.2 m. Calculate the angular velocity of the cylinder at the instant when the glider has a speed of 27 m s^{-1}. Include an appropriate unit for your answer. [2]

e. After takeoff the cable is released and the unpowered glider moves horizontally at constant speed. The wings of the glider provide a lift force. The diagram shows the lift force acting on the glider and the direction of motion of the glider. [2]



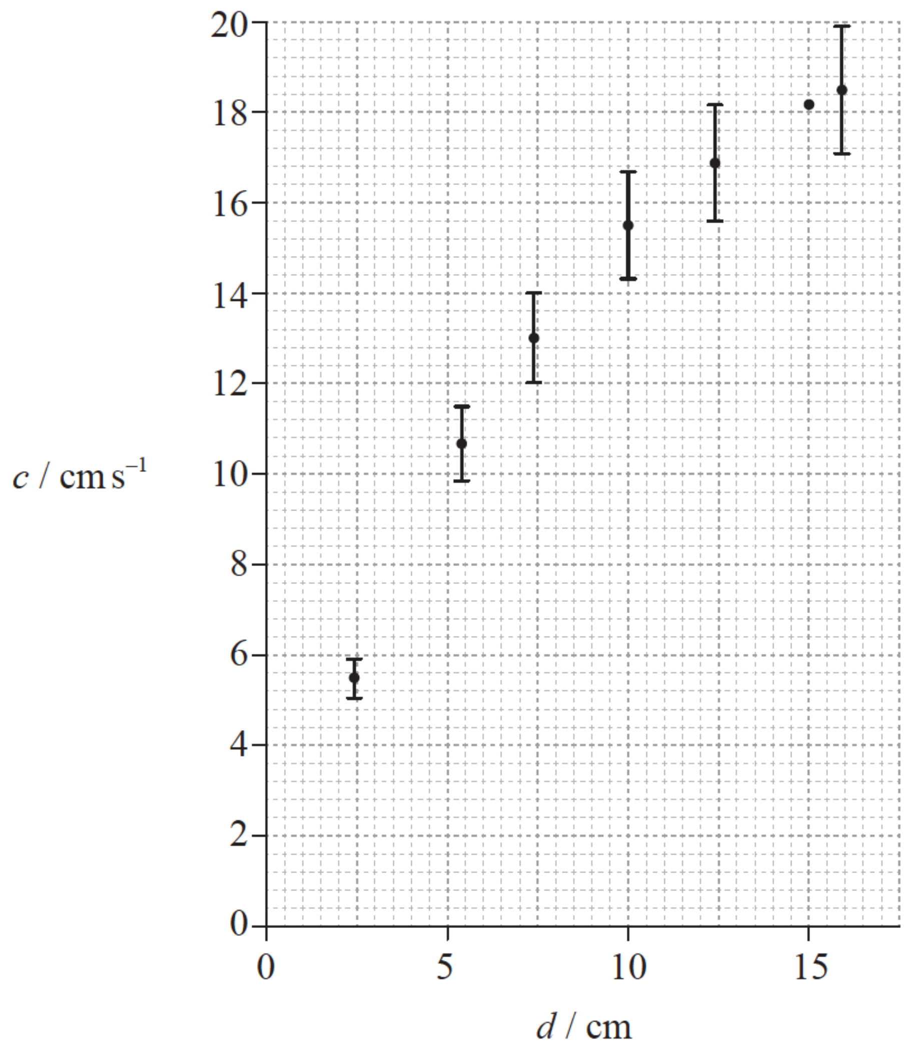
Draw the forces acting on the glider to complete the free-body diagram. The dotted lines show the horizontal and vertical directions.

f. Explain, using appropriate laws of motion, how the forces acting on the glider maintain it in level flight. [2]

g. At a particular instant in the flight the glider is losing 1.00 m of vertical height for every 6.00 m that it goes forward horizontally. At this instant, the horizontal speed of the glider is 12.5 m s^{-1}. Calculate the **velocity** of the glider. Give your answer to an appropriate number of significant figures. [3]

Data analysis question.

Caroline carried out an experiment to measure the variation with water depth *d* of the wave speed *c* of a surface water wave. Her data are shown plotted below.



The uncertainty in the water depth *d* is too small to be shown. Uncertainties in the measurement of the wave speed *c* are shown as error bars on the graph except for the data point corresponding to *d*=15 cm.

a. Caroline calculated the wave speed by measuring the time *t* for the wave to travel 150 cm. The uncertainty in this distance is 2 cm. For the reading at a water depth of 15 cm, the time *t* is 8.3 s with an uncertainty 0.5 s. [4]

(i) Show that the absolute uncertainty in the wave speed at this time is 1.3 cm s^{-1}.

(ii) On the graph opposite, draw the error bar for the data point corresponding to *d*=15 cm.

b. Caroline hypothesized that the wave speed *c* is directly proportional to the water depth *d*. [3]

(i) On the graph opposite, draw a line of best-fit for the data.
(ii) Suggest if the data support this hypothesis.

c. Another student proposes that *c* is proportional t o $d^{0.5}$. [1]

State a suitable graph that can be plotted to test this proposal.

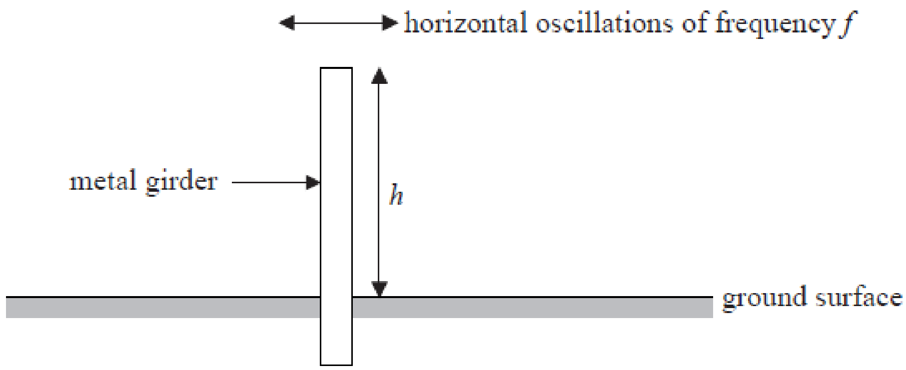
d. There is a systematic error in Caroline's determination of the depth. [2]

(i) State what is meant by a systematic error.

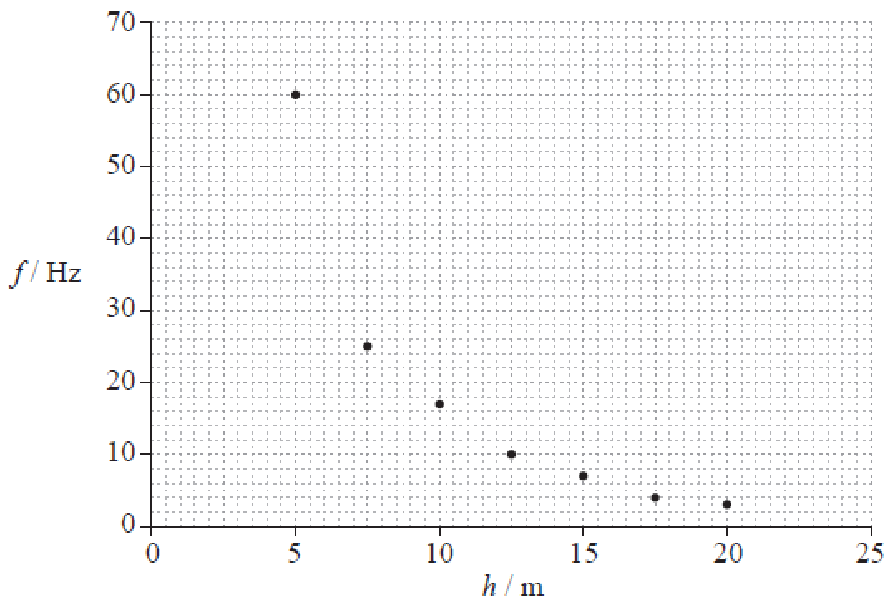
(ii) State how the graph in (c) would indicate that there is a systematic error.

Data analysis question.

Metal girders are often used in buildings that have been constructed to withstand earthquakes. To aid the design of these buildings, experiments are undertaken to measure how the natural frequency f of horizontal oscillations of metal girders varies with their dimensions. In an experiment, f was measured for vertically supported girders of the same cross-sectional area but with different heights h.



The graph shows the plotted data for this experiment. Uncertainties in the data are not shown.



a. Draw a best-fit line for the data. [1]

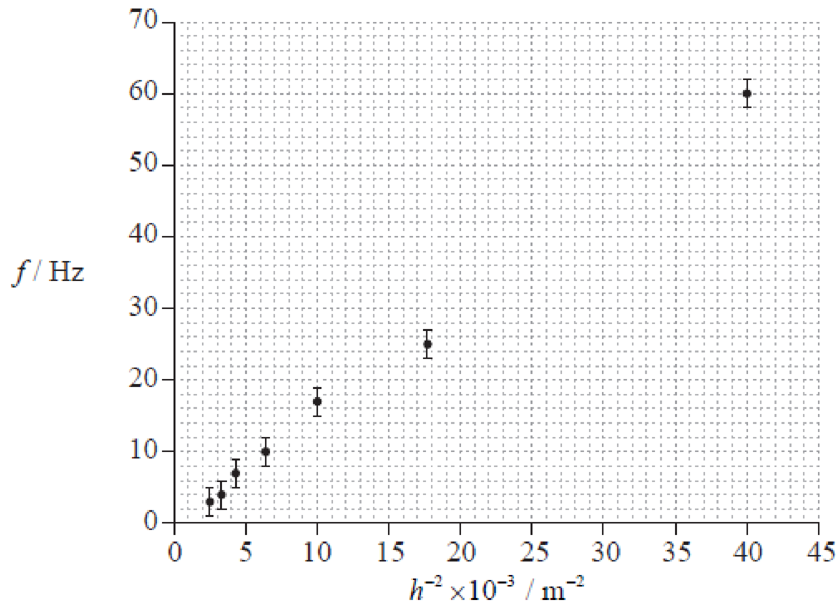
b. It is hypothesized that the frequency f is inversely proportional to the height h. [4]

By choosing **two** well separated points on the best-fit line that you have drawn in (a), show that this hypothesis is incorrect.

c. Another suggestion is that the relationship between f and h is of the form shown below, where k is a constant. [5]

$$f = \frac{k}{h^2}$$

The graph shows a plot of f against h^{-2}.



The uncertainties in h^{-2} are too small to be shown.

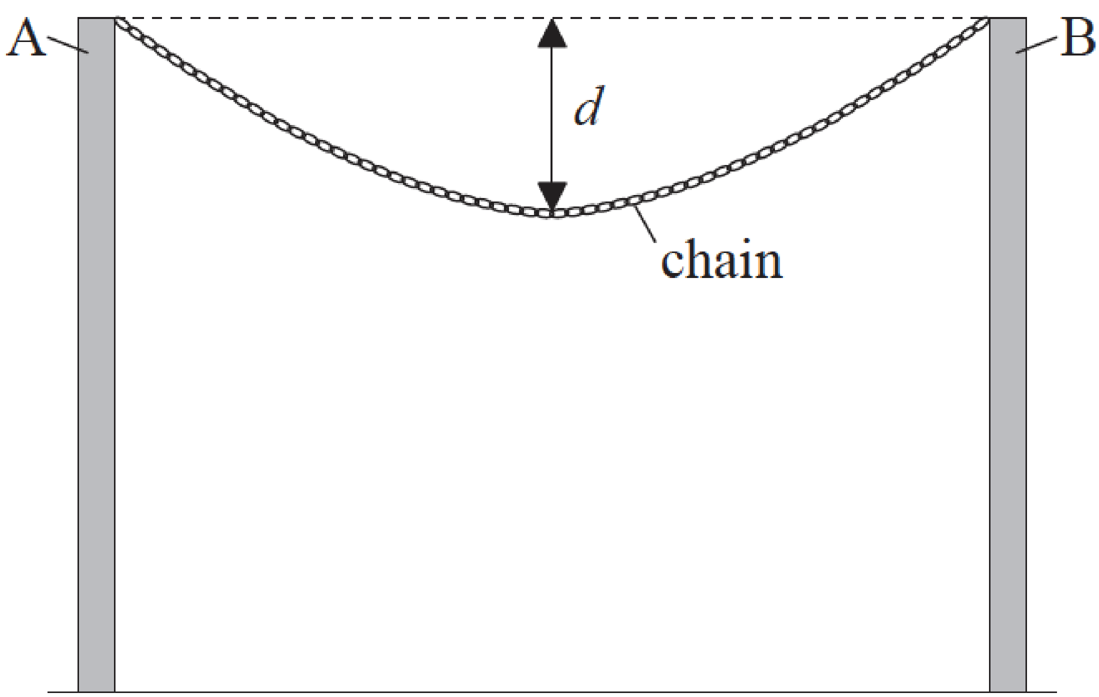
(i) Draw a best-fit line for the data that supports the relationship $f = \frac{k}{h^2}$.

(ii) Determine, using the graph, the constant k.

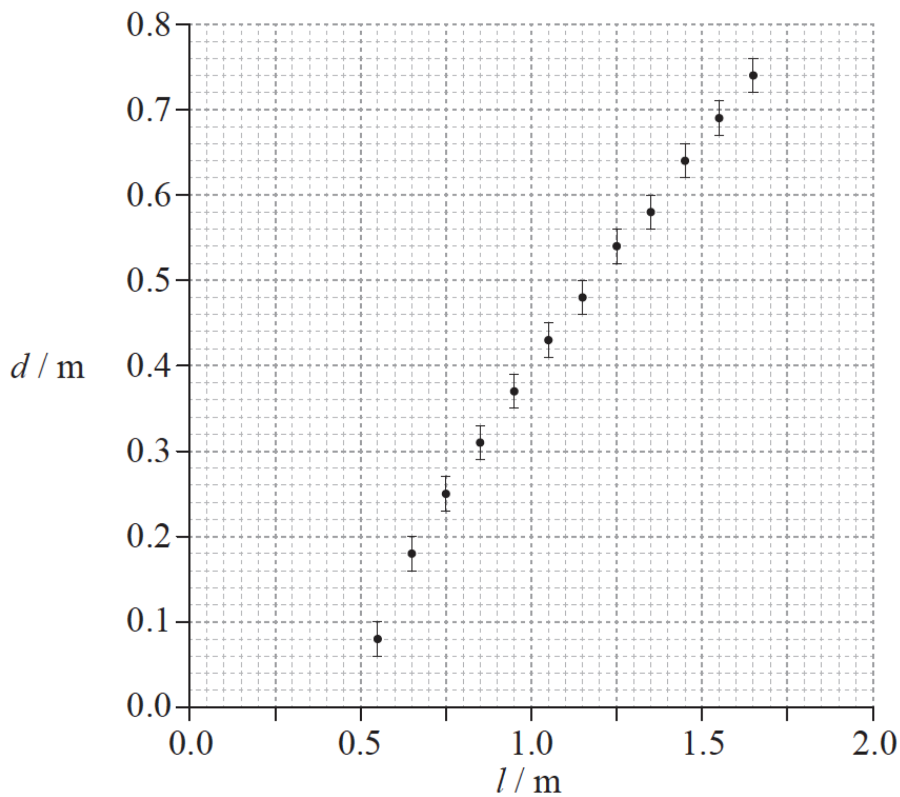
d. State **one** reason why the results of the experiment could not be used to predict the natural frequency of oscillation for girders of height 50 m. [1]

Data analysis question.

A chain is suspended between two vertical supports A and B. The chain is made of a number of identical metal links.



The length l of the chain can be increased by adding extra links. An experiment was undertaken to investigate how the sag d of the midpoint of the chain, measured from the horizontal between A and B, varies with l. The data obtained are shown plotted below. The uncertainties in l are too small to be shown.



a. Draw a best-fit line for the data points on the graph opposite. [1]

b. With reference to your answer to (a), [4]

(i) explain why the relationship between *d* and *l* is not linear.

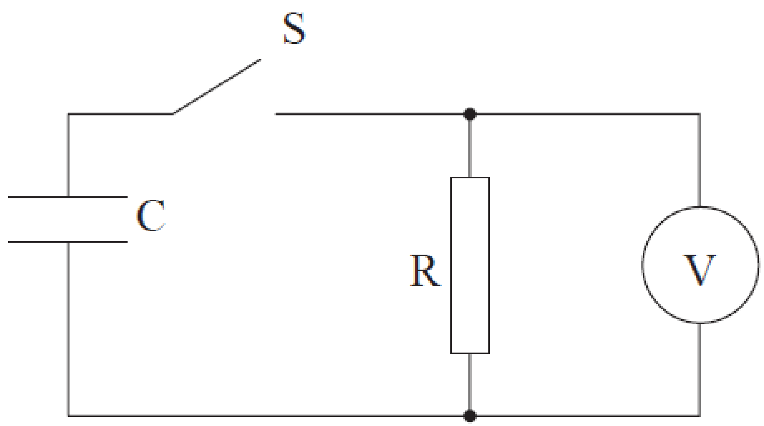
(ii) estimate the horizontal distance between the supports A and B.

c. Before the experiment was carried out, it was hypothesized that *d* depends on $\sqrt{l}$. Determine, using your answer to (a), whether this hypothesis is valid. [4]

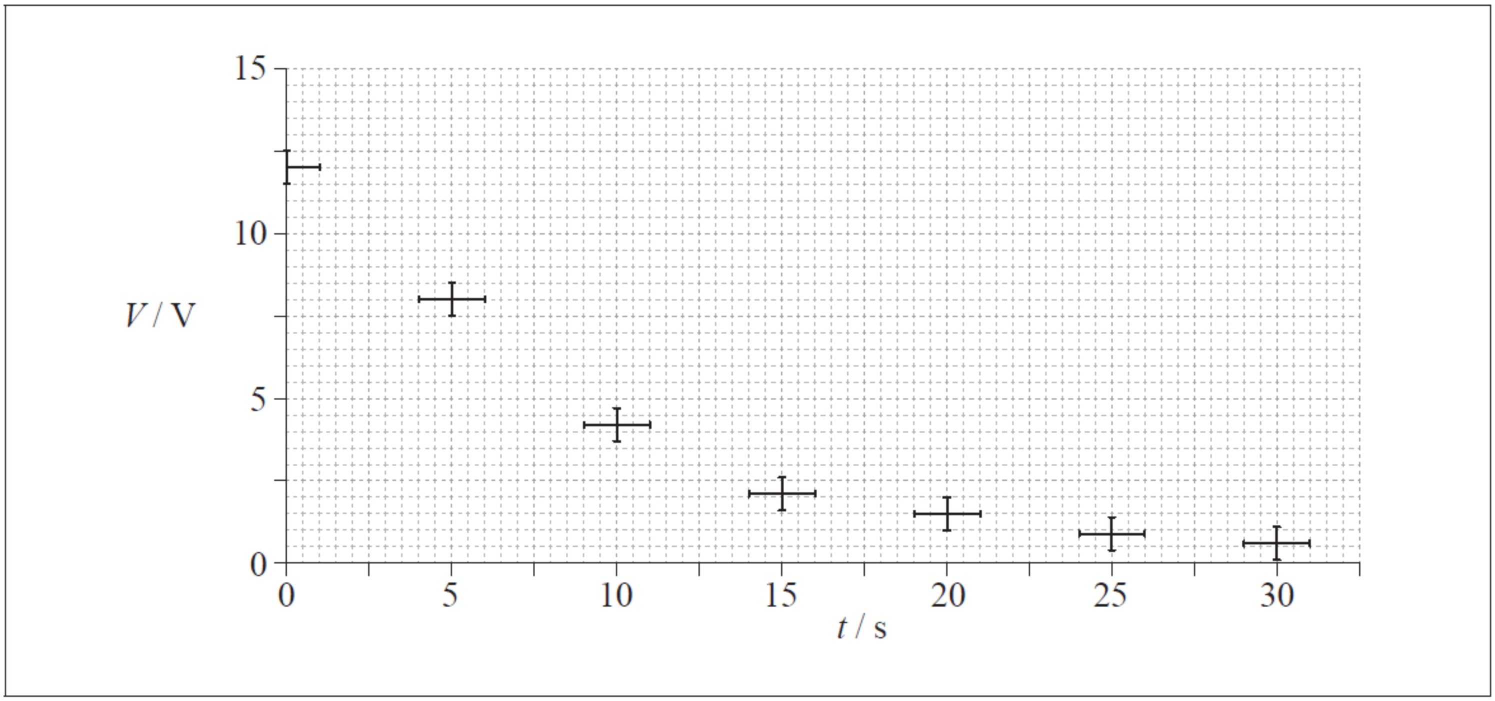
Data analysis question.

A capacitor is a device that can be used to store electric charge.

a. An experiment was undertaken to investigate one of the circuit properties of a capacitor. A capacitor C was connected via a switch S to a resistance R and a voltmeter V. [6]



The initial potential difference across C was 12V. The switch S was closed and the potential difference V across R was measured at various times t. The data collected, along with error bars, are shown plotted below.



(i) On the graph opposite, draw a best-fit line for the data starting from t = 0.

(ii) It was hypothesized that the decay of the potential difference across the capacitor is exponential. Determine, using the graph, whether this hypothesis is true **or** not.

b. The time constant τ of the circuit is defined as the time it would take for the capacitor to discharge were it to keep discharging at its initial rate. [3]

Use the graph in (a) to calculate the

(i) initial rate of decay of potential difference V.

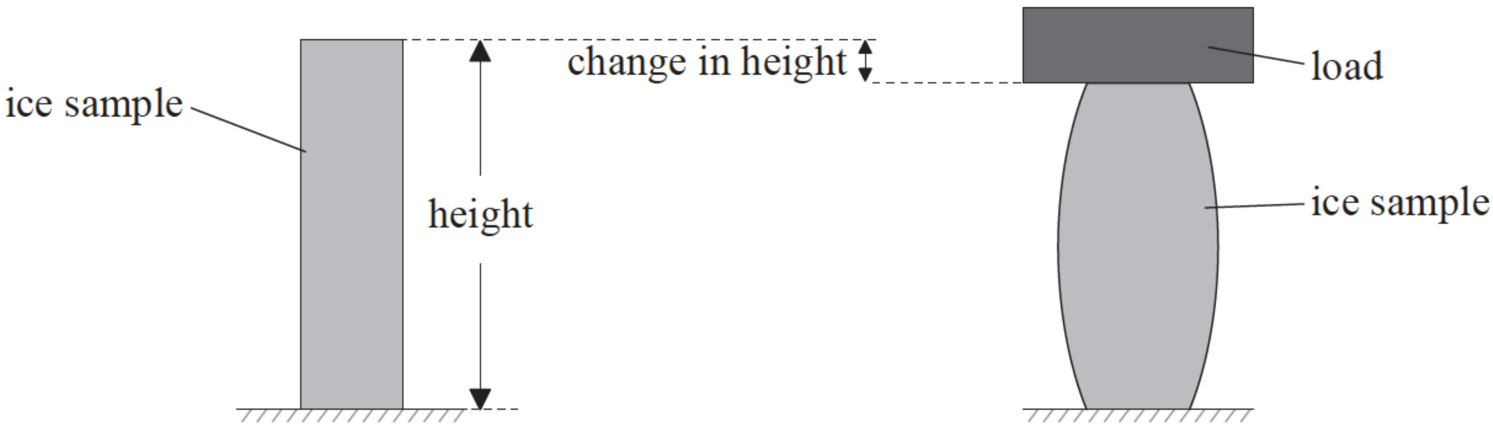
(ii) time constant τ.

c. The time constant $\tau = RC$ where R is the resistance and C is a property called capacitance. The effective resistance in the circuit is 10 MΩ. [1]

Calculate the capacitance C.

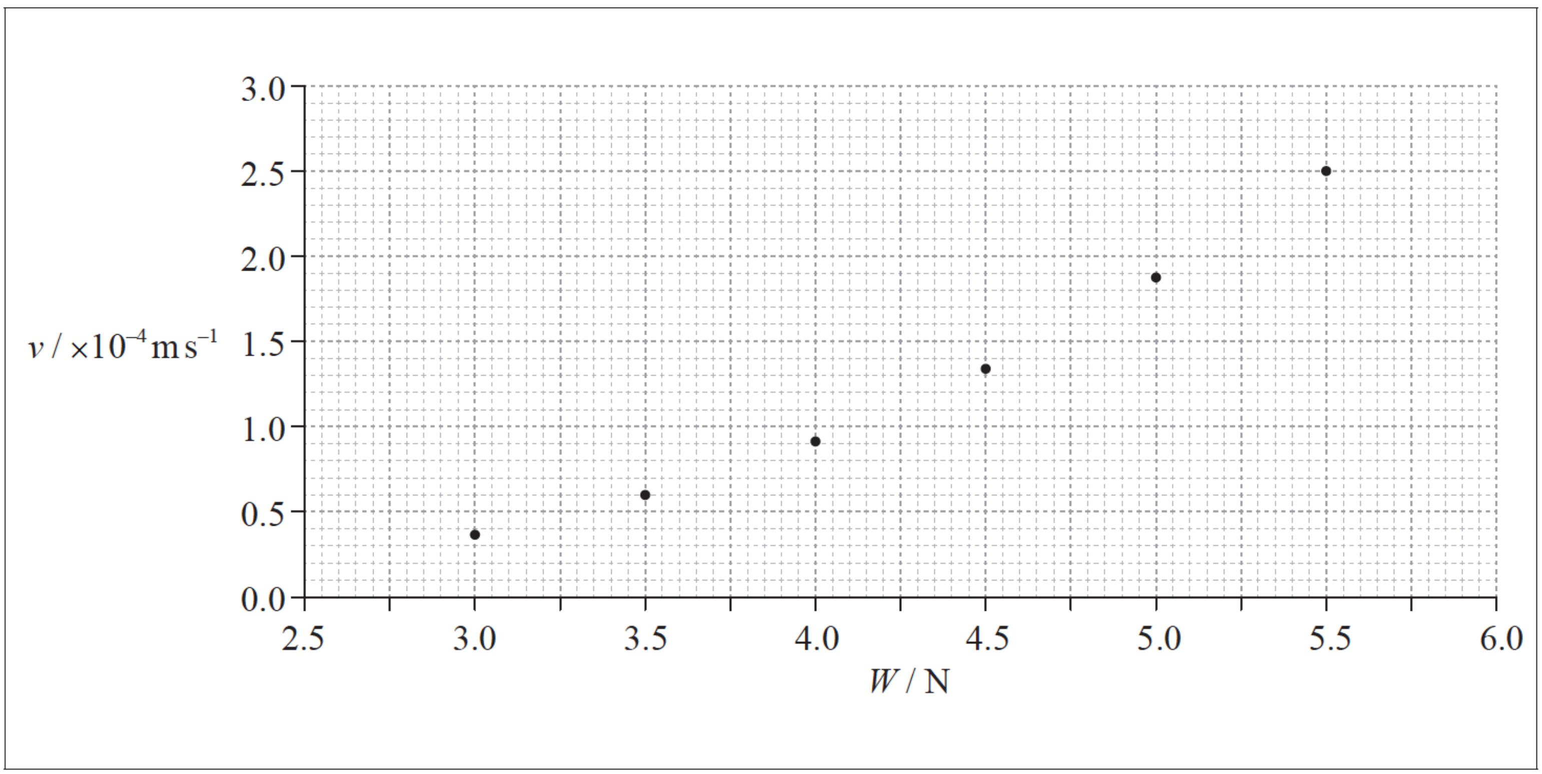
Data analysis question.

The movement of glaciers can be modelled by applying a load to a sample of ice.



After the load has been applied, it is observed to move downwards at a constant speed *v* as the ice deforms. The constant speed *v* is measured for different loads. The graph shows the variation of *v* with load *W* for a number of identical samples of ice.

The data points are plotted below.



The uncertainty in *v* is ±20 μm s^{-1} and the uncertainty in *W* is negligible.

a. (i) On the graph opposite, draw error bars on the first and last points to show the uncertainty in *v*. [2]

(ii) On the graph opposite, draw the line of best-fit for the data points.

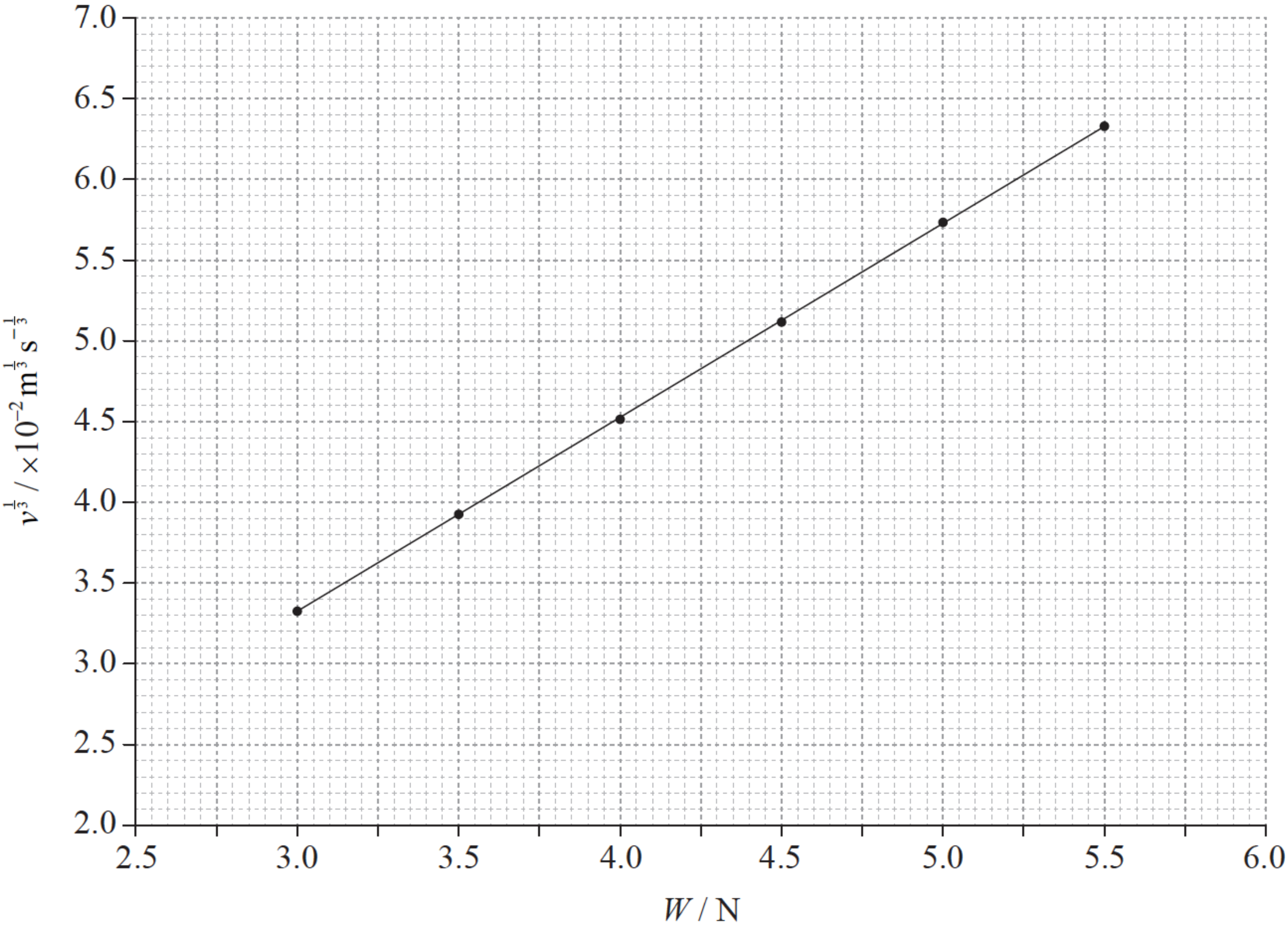
b. Explain whether the data support the hypothesis that v is directly proportional to *W*. [1]

c. Theory suggests that the relation between *v* and *W* is [3]

$$v = kW^3$$

where *k* is a constant.

To test this hypothesis a graph of $v^{\frac{1}{3}}$ against W is plotted.



At W=5.5 N the speed is 250±20 μm s^{-1}.

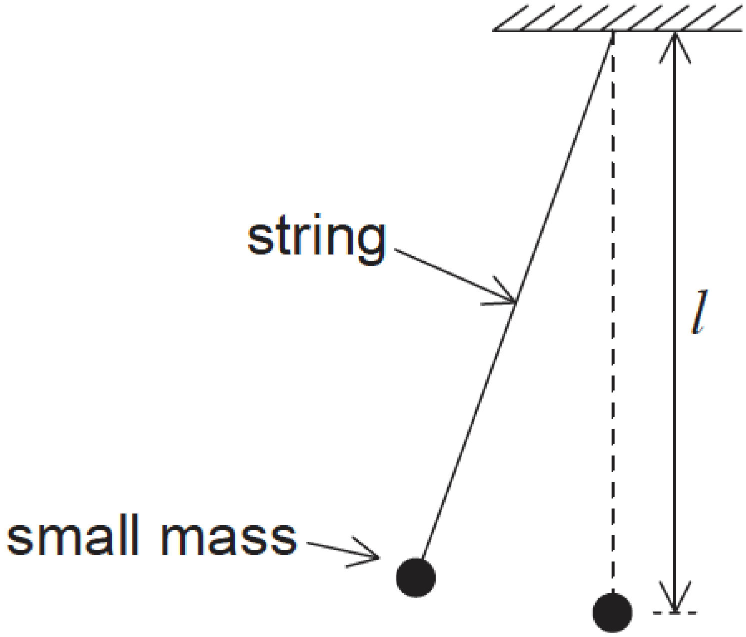
Calculate the uncertainty in $v^{\frac{1}{3}}$ for a load of 5.5 N.

d. (i) Using the graph in (c), determine *k* without its uncertainty. [5]

(ii) State an appropriate unit for your answer to (d)(i).

Data analysis question.

A simple pendulum of length *l* consists of a small mass attached to the end of a light string.



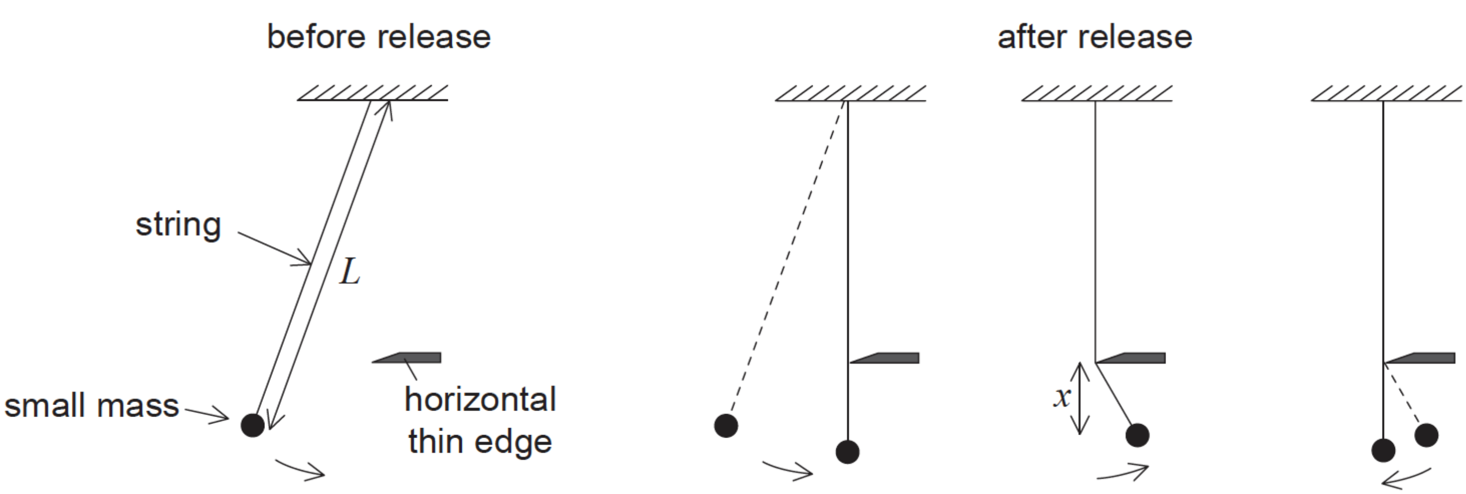
The time *T* taken for the mass to swing through one cycle is given by

$$T = 2\pi\sqrt{\frac{l}{g}}$$

where *g* is the acceleration due to gravity.

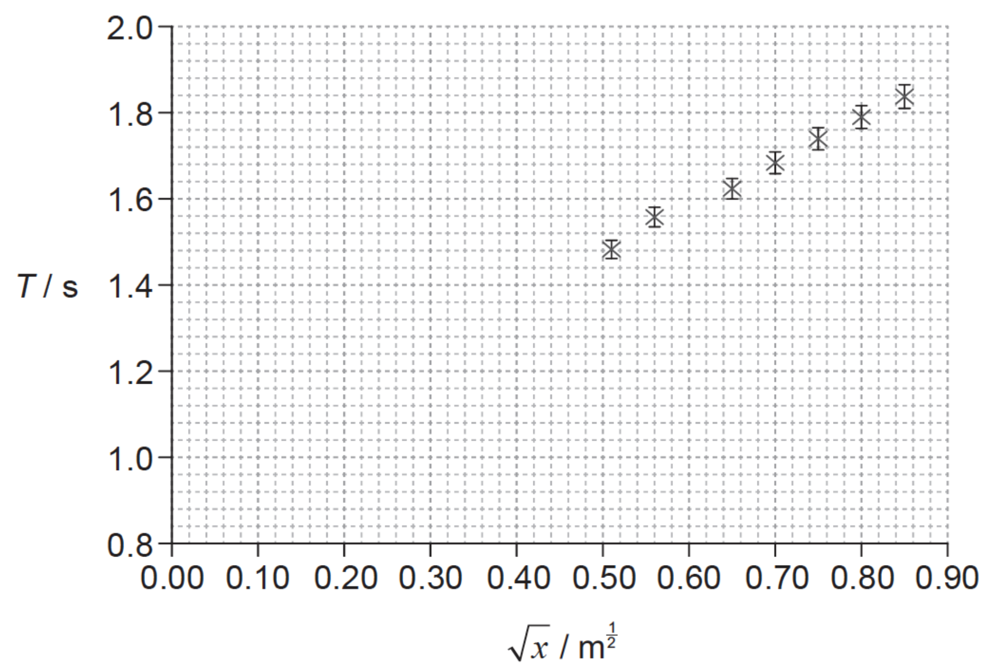
a. A student measures *T* for one length *l* to determine the value of *g*. Time $T = 1.9s \pm 0.1s$ and length $l = 0.880m \pm 0.001m$. Calculate the fractional uncertainty in *g*. [2]

b. The student modifies the simple pendulum of length *L* so that, after release, it swings for a quarter of a cycle before the string strikes a horizontal thin edge. For the next half cycle, the pendulum swings with a shorter length *x*. The string then leaves the horizontal thin edge to swing with its original length *L*. [9]



The length *L* of the string is kept constant during the experiment. The vertical position of the horizontal thin edge is varied to change *x*.

The graph shows the variation of the time period with $\sqrt{x}$ for data obtained by the student together with error bars for the data points. The error in $\sqrt{x}$ is too small to be shown.



(i) Deduce that the time period for one complete oscillation of the pendulum is given by

$$T = \frac{\pi}{\sqrt{g}}\left(\sqrt{L} + \sqrt{x}\right)$$

(ii) On the graph, draw the best-fit line for the data.

(iii) Determine the gradient of the graph.

(iv) State the value of the intercept on the *T*-axis.

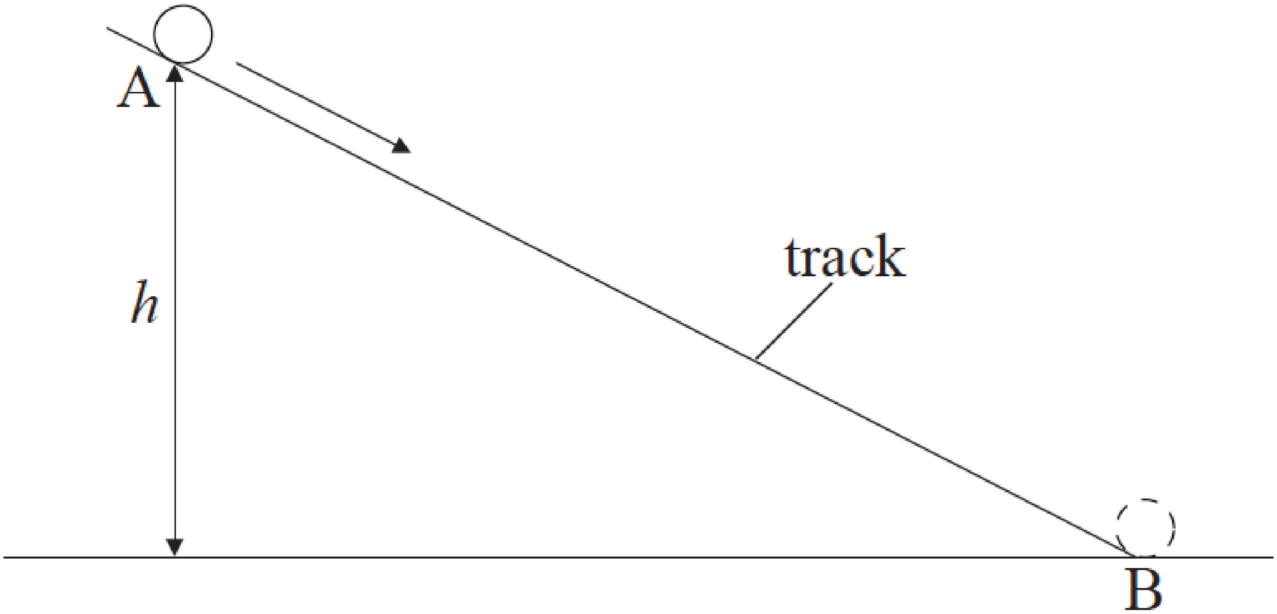
(v) The equation of a straight line is $y = mx + c$. Determine, using your answers to (b)(iii) and (b)(iv), the intercept on the $\sqrt{x}$-axis.

(vi) Calculate *L*.

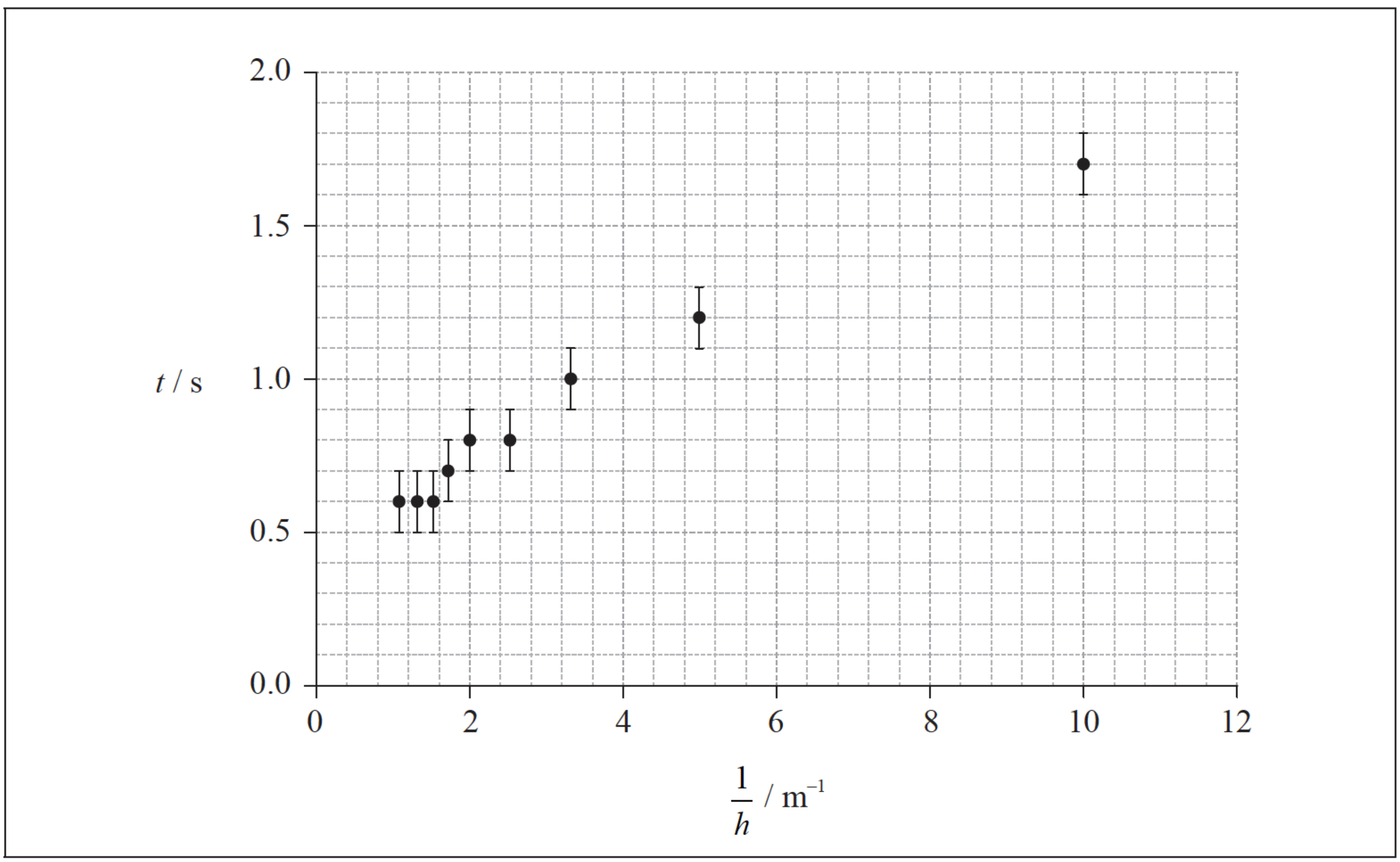
Data analysis question.

A small sphere rolls down a track of constant length AB. The sphere is released from rest at A.

The time *t* that the sphere takes to roll from A to B is measured for different values of height *h*.



A student suggests that t is proportional to $\frac{1}{h}$. To test this hypothesis a graph of t against $\frac{1}{h}$ is plotted as shown on the axes below. The uncertainty in t is shown and the uncertainty in $\frac{1}{h}$ is negligible.



a. (i) Draw the straight line that best fits the data. [2]

(ii) State why the data do not support the hypothesis.

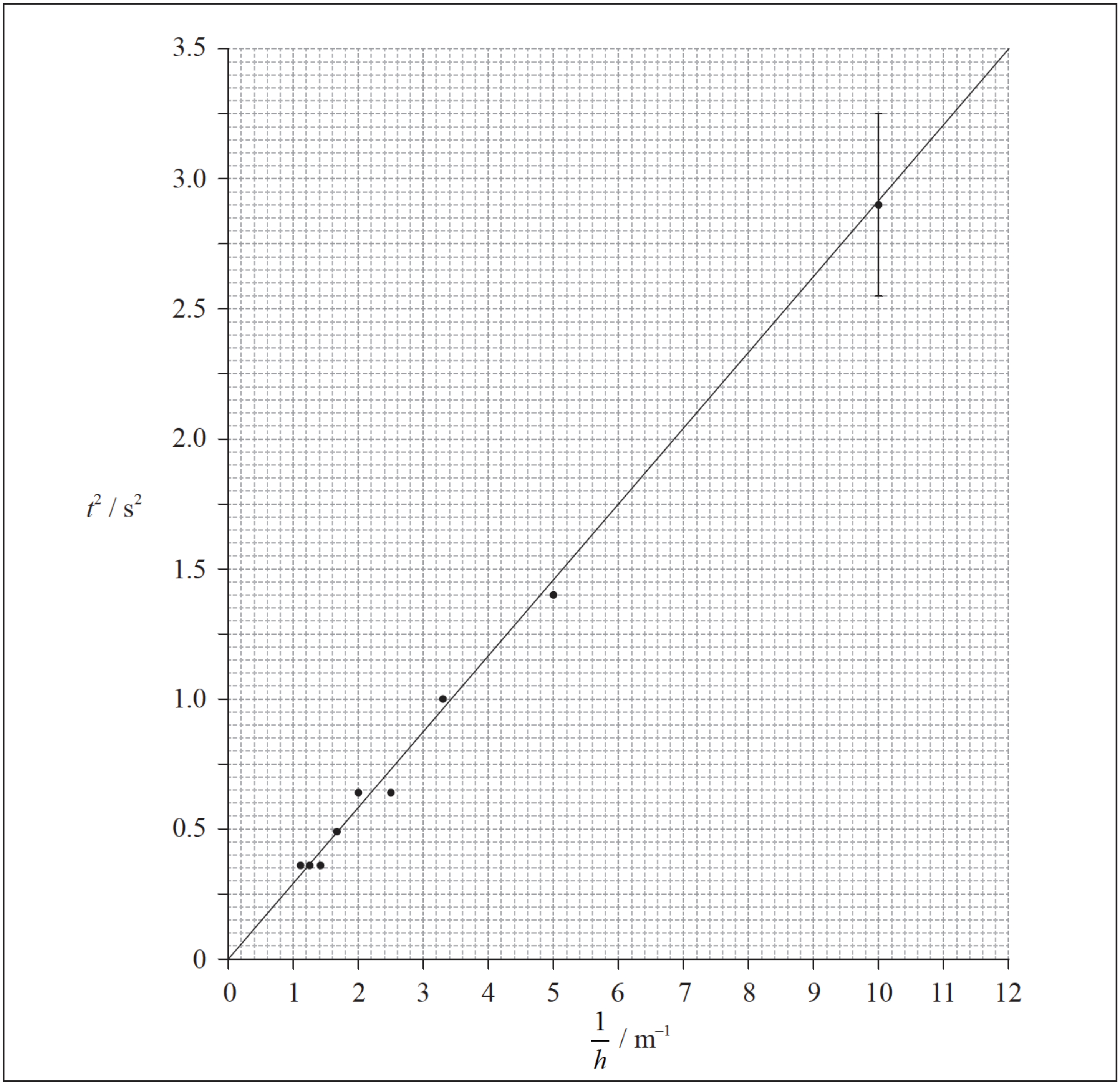
b. Another student suggests that the relationship between t and h is of the form [9]

$$t = k\sqrt{\frac{1}{h}}$$

where *k* is a constant.

To test whether or not the data support this relationship, a graph of t^2 against $\frac{1}{h}$ is plotted as shown below.

The best-fit line takes into account the uncertainties for all data points.



The uncertainty in t^2 for the data point where $\frac{1}{h} = 10.0\text{m}^{-1}$ is shown as an error bar on the graph.

(i) State the value of the uncertainty in t^2 for $\frac{1}{h} = 10.0\text{m}^{-1}$.

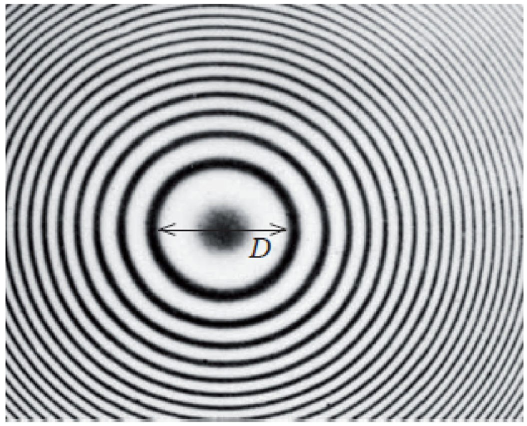
(ii) Calculate the uncertainty in t^2 when $t = 0.8 \pm 0.1$s. Give your answer to an appropriate number of significant digits.

(iii) Use the graph to determine the value of *k*. Do not calculate its uncertainty.

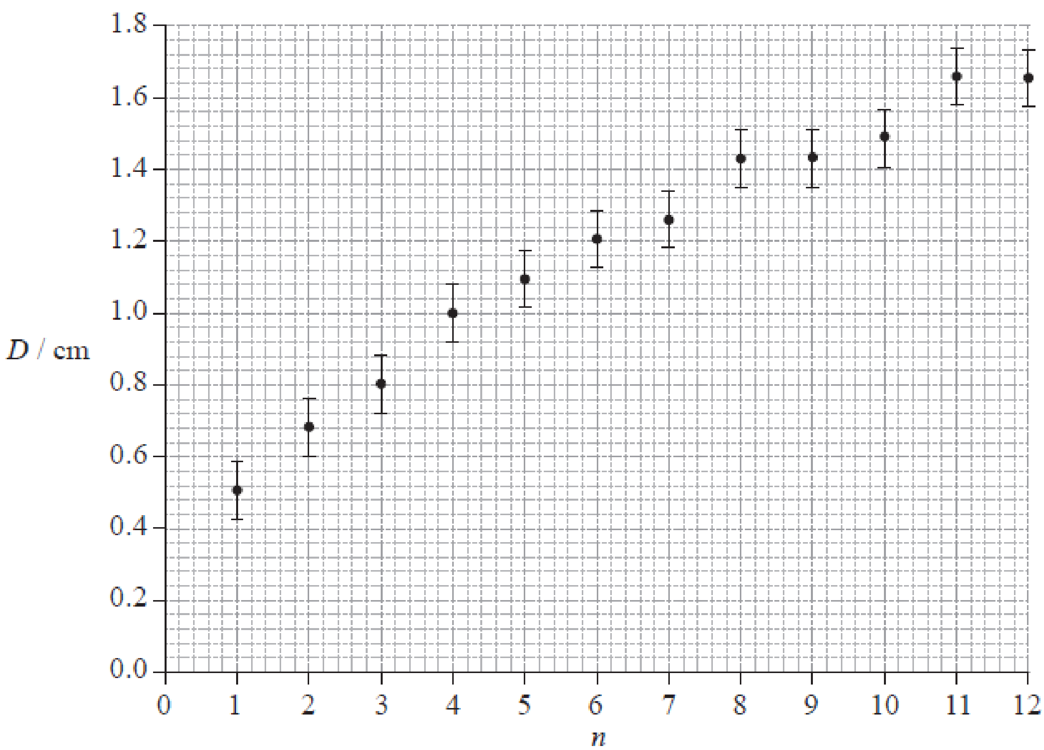
(iv) State the unit of *k*.

Data analysis question.

The photograph below shows a magnified image of a dark central disc surrounded by concentric dark rings. These rings were produced as a result of interference of monochromatic light.



The graph below shows how the ring diameter *D* varies with the ring number *n*. The innermost ring corresponds to *n* = 1. The corresponding diameter is labelled in the photograph. Error bars for the diameter *D* are shown.

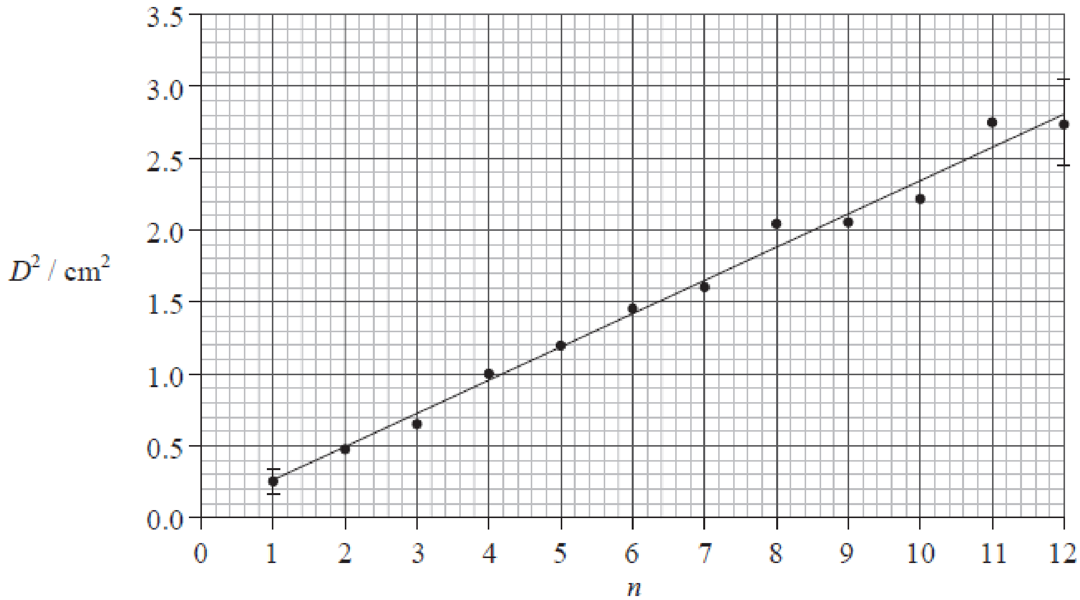


a. State **one** piece of evidence that shows that *D* is not proportional to *n*. [1]

b. On the graph opposite, draw the line of best-fit for the data points. [2]

c. Theory suggests that $D^2 = kn$. [8]

A graph of D^2 against *n* is shown below. Error bars are shown for the first and last data points only.



(i) Using the graph on page 2, calculate the percentage uncertainty in D^2, of the ring n = 7.

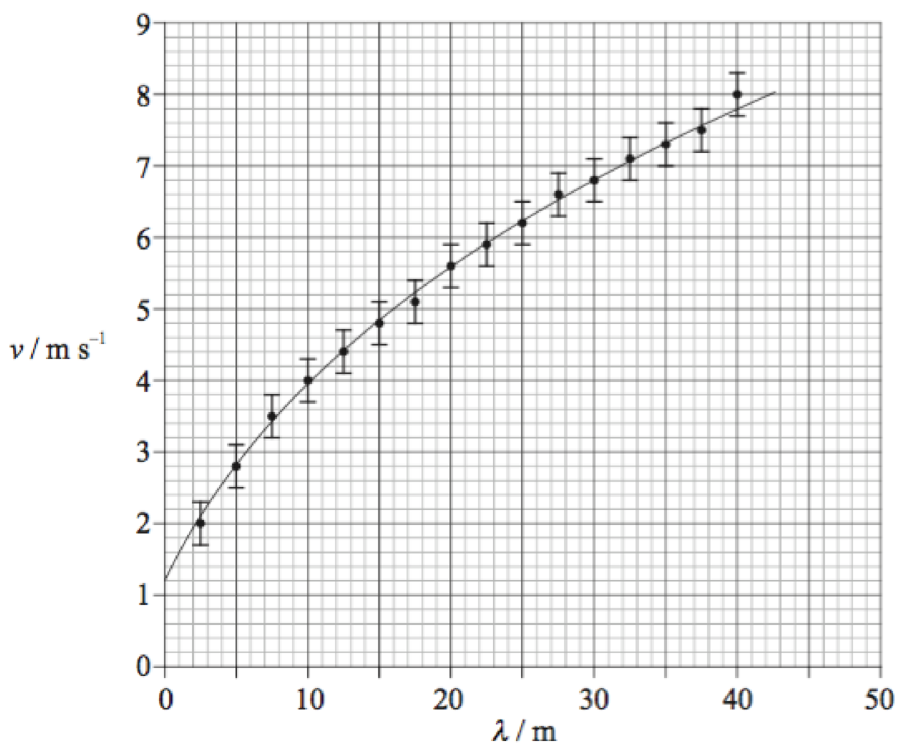
(ii) Based on the graph opposite, state **one** piece of evidence that supports the relationship $D^2 = kn$.

(iii) Use the graph opposite to determine the value of the constant k, as well as its uncertainty.

(iv) State the unit for the constant k.

Data analysis question.

The speed v of waves on the surface of deep water depends only on the wavelength λ of the waves. The data gathered from a particular region of the Atlantic Ocean are plotted below.



The uncertainty in the speed v is ±0.30 m s^{-1} and the uncertainty in λ is too small to be shown on the diagram.

State, with reference to the graph,

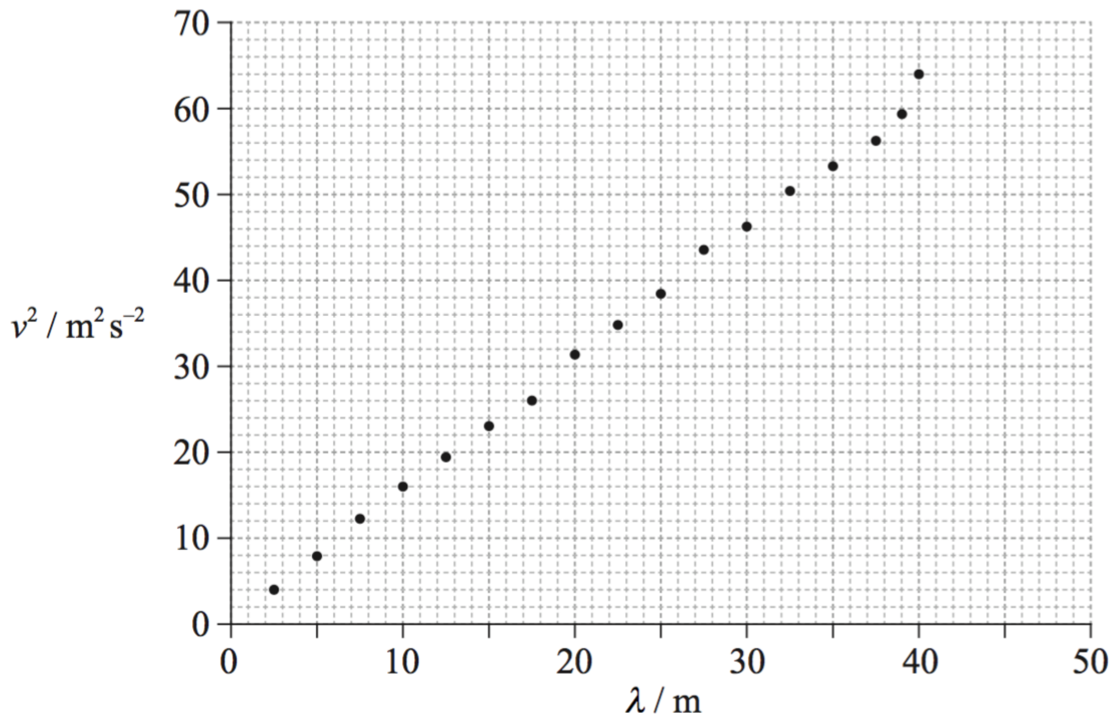
a. (i) why v is not directly proportional to λ. [2]

(ii) the value of v for λ=39m.

b. It is suggested that the relationship between v and λ is of the form [9]

$$v = a\sqrt{\lambda}$$

where a is a constant. To test the validity of this hypothesis, values of v^2 against λ are plotted below.



(i) Use your answer to (a)(ii) to show that the absolute uncertainty in v^2 for a wavelength of 39 m is ±5 m^2 s^{-2}.

(ii) The absolute uncertainty in v^2 for a wavelength of 2.5 m is ± 1m^2 s^{-2}. Using this value and the value in (b)(i), construct error bars for v^2 at the data points for λ=2.5 m and 39 m.

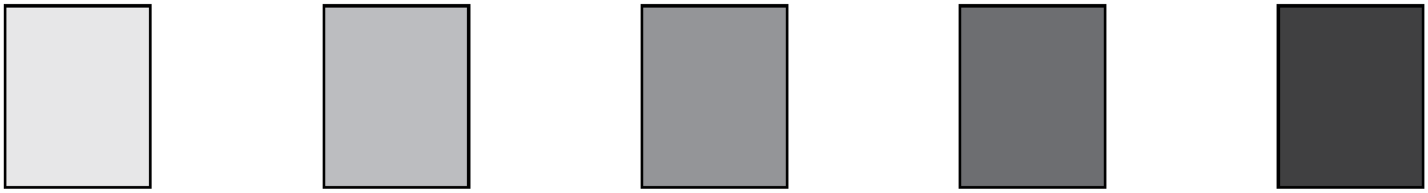
(iii) State why the plotted data in (b)(ii) suggest that it is likely that v is proportional to $\sqrt{\lambda}$.

(iv) Use the graph opposite to determine the constant a.

(v) Theory shows that $a = \sqrt{\frac{k}{2\pi}}$. Determine a value for k.

Data analysis question.

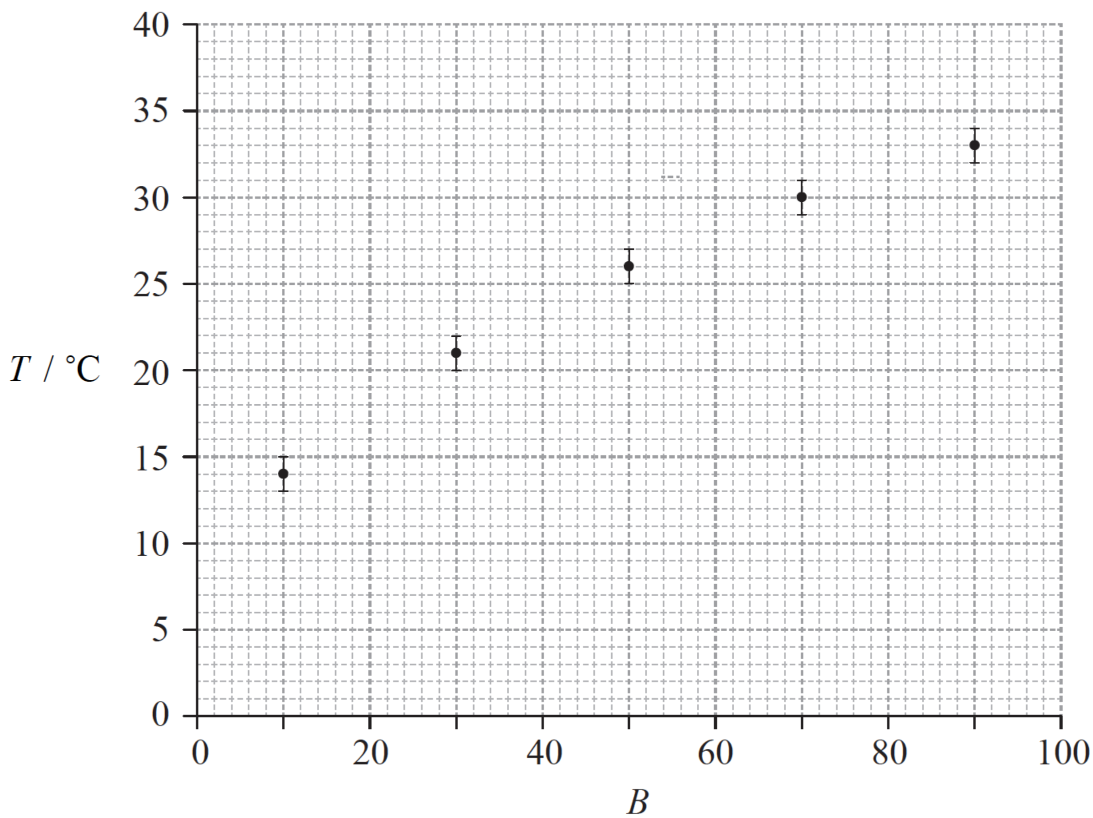
Connie and Sophie investigate the effect of colour on heat absorption. They make grey paint by mixing black and white paint in different ratios. Five identical tin cans are painted in five different shades of grey.



10 % black paint 30 % black paint 50 % black paint 70 % black paint 90 % black paint

Connie and Sophie put an equal amount of water at the same initial temperature into each can. They leave the cans under a heat lamp at equal distances from the lamp. They measure the temperature increase of the water, T, in each can after one hour.

a. Connie suggests that T is proportional to B, where B is the percentage of black in the paint. To test this hypothesis, she plots a graph of T against B, as shown on the axes below. The uncertainty in T is shown and the uncertainty in B is negligible. [6]



(i) State the value of the absolute uncertainty in T.

(ii) Comment on the fractional uncertainty for the measurement of T for B=10 and the measurement of T for B=90.

(iii) On the graph opposite, draw a best-fit line for the data.

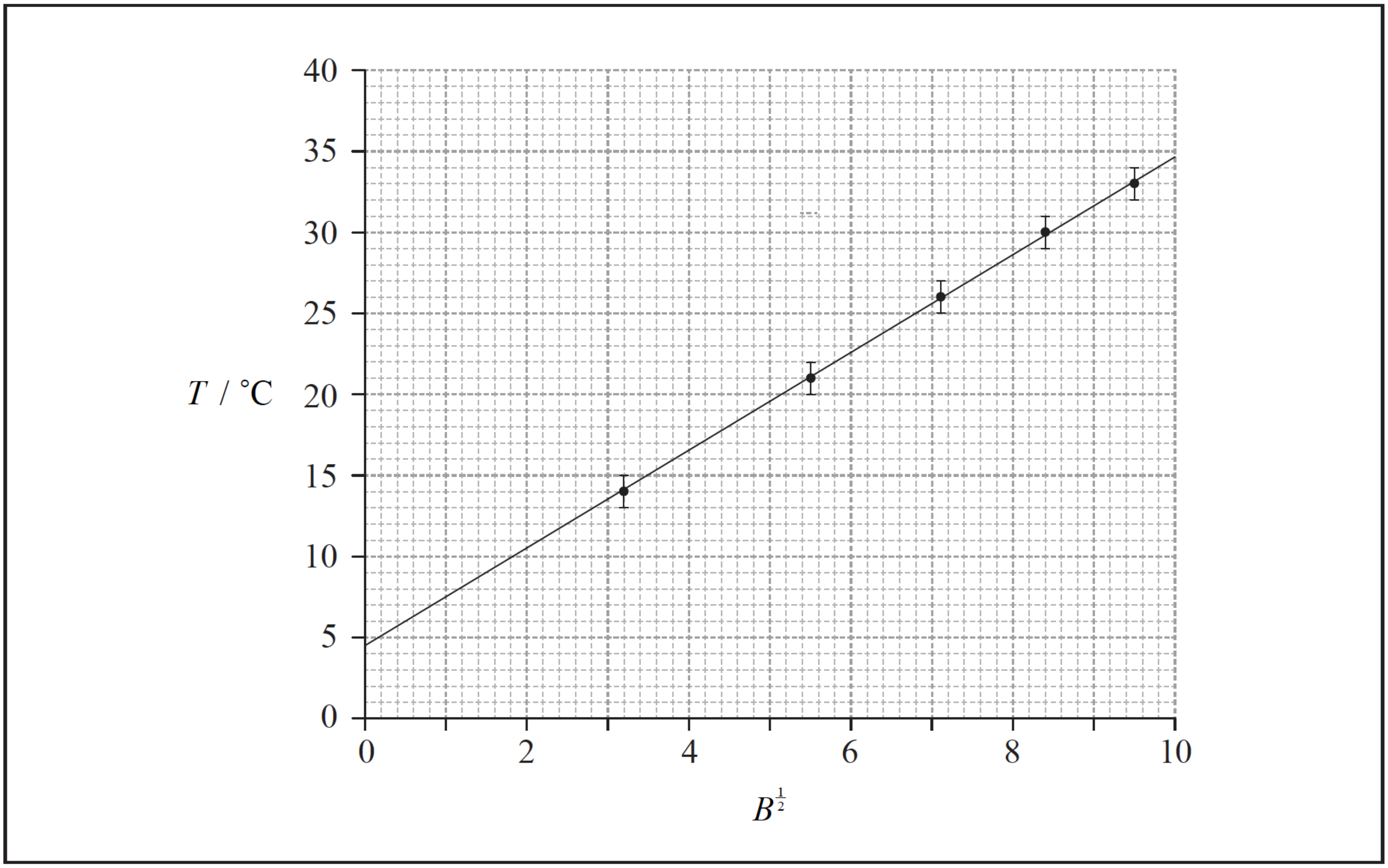
(iv) Outline why the data do not support the hypothesis that T is proportional to B.

b. Sophie suggests that the relationship between T and B is of the form [5]

$$T = kB^{\frac{1}{2}} + c$$

where k and c are constants.

To test whether or not the data support this relationship, a graph of T against $B^{\frac{1}{2}}$ is plotted as shown below. The uncertainty in T is shown and the uncertainty in $B^{\frac{1}{2}}$ is negligible.

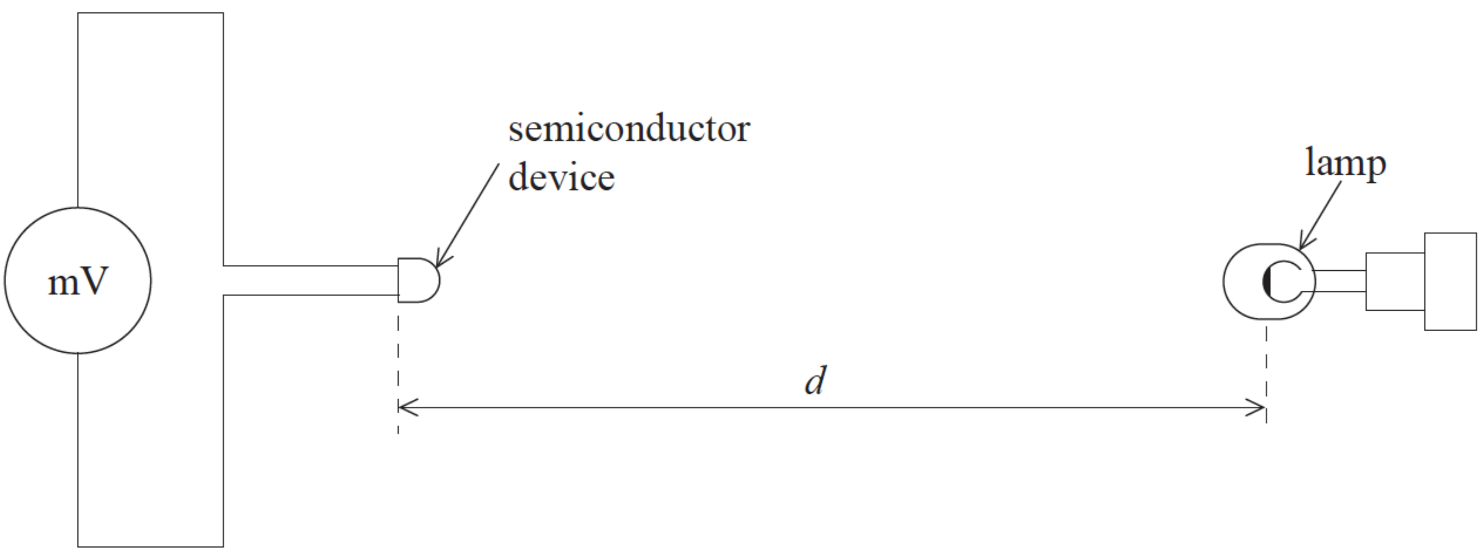


(i) Use the graph to determine the value of *c* with its uncertainty.

(ii) State the unit of *k*.

Data analysis question.

A particular semiconductor device generates an emf, which varies with light intensity. The diagram shows the experimental arrangement which a student used to investigate the variation with distance *d* of the emf ε. The power output of the lamp was constant. (The power supply for the lamp is not shown.)



The table shows how ε varied with *d*.

d / cm	ε / mV
19.1	5.5
18.0	6.0
16.0	8.6
14.0	11.9
12.0	19.7
10.0	37.5

a. Outline why the student has recorded the ε values to different numbers of significant digits but the same number of decimal places. [2]

b. On looking at the results the student suggests that ε could be inversely proportional to d. He proceeds to multiply each d value by the corresponding value of ε. [5]

(i) Explain why this procedure can be used to disprove the student's suggestion but it cannot prove it.

(ii) Using the data for d values of 19.1 cm, 16.0 cm and 10.0 cm discuss whether or not ε is inversely proportional to d.

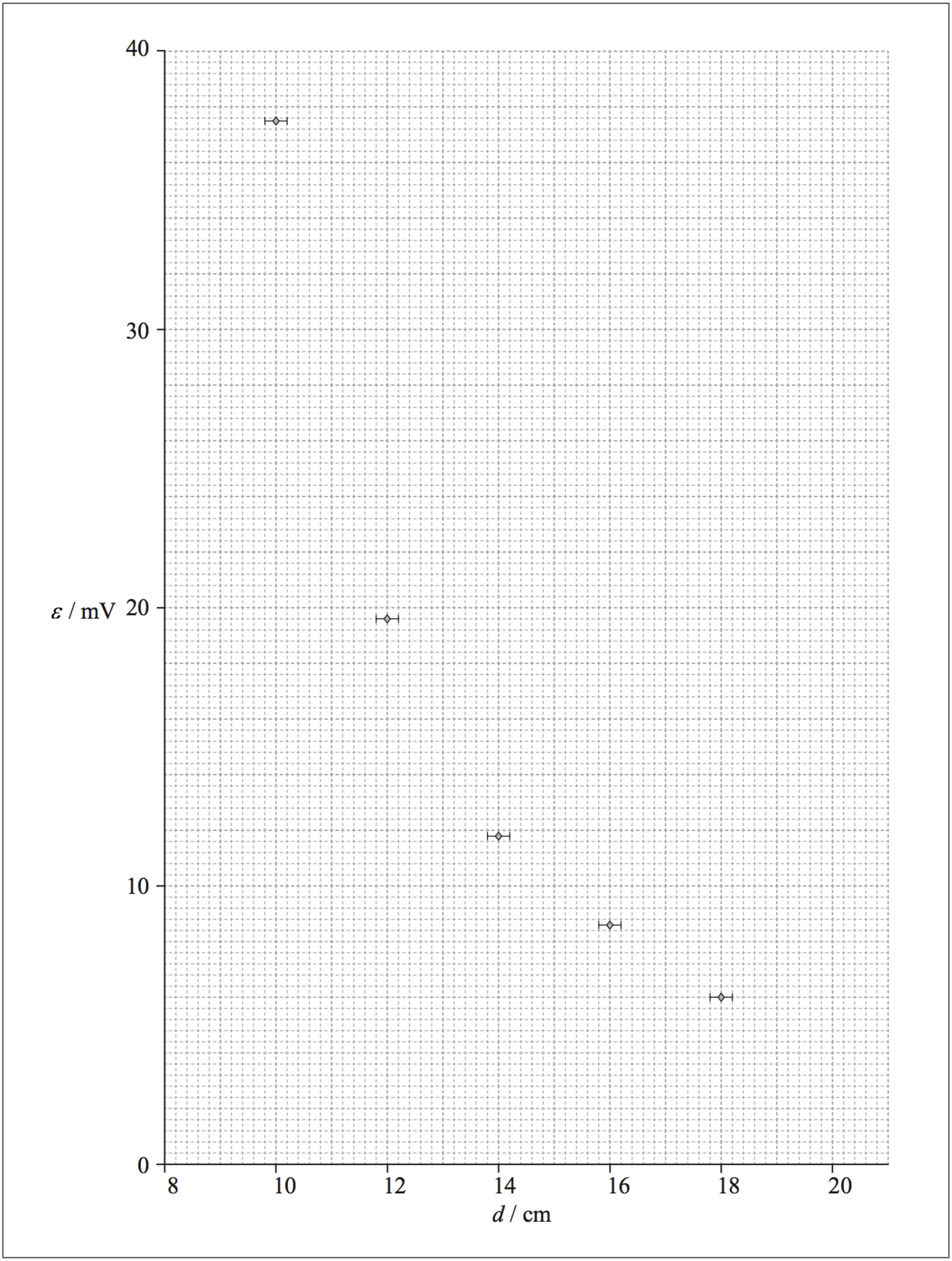
c. The graph shows some of the data points with the uncertainty in the d values. [3]

On the graph

(i) draw the data point corresponding to the value of d=19.1 cm.

(ii) assuming that there is a constant absolute uncertainty in measuring all values of d, draw the error bar for the data point in (c)(i).

(iii) sketch the line of best-fit for all the plotted points.



40
30
ε / mV
20
10
0
8
10
12
14
16
18
20
d / cm

d. All values of ε have a percentage uncertainty of $\pm 3\%$. Calculate the percentage uncertainty in the product $d\varepsilon$ for the value of $d = 18.0\text{cm}$. [2]